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NOTE ON THE USE OF CONICAL SOURCE GEOMETRY IN THE HOMOGENEOUS FIELD AXIAL FOCUSING MAGNETIC β-RAY SPECTROMETER

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Late in 1951 D. E. Muller proposed the idea of a small right circular cone as an advantageous shape on which to deposit β-active sources, to be used for the precision measurement of conversion line energies in the axial focusing homogeneous magnetic field β-ray spectrometer[1,2] recently developed at this Institute.

Referring to Fig. 1, Muller's idea was to make the semi-apex angle, α_c, of the conical source equal to α_1, the angle of inclination of the "ultimate β-ray traces" with the spectrometer axis at the point P where these traces just pass over the inner jaw of the annular resolving slit. The "β-ray traces" are sections by an axial plane of the surfaces (sinuosoids of revolution) on which lie a family of helical β-ray trajectories emanating with specified kinetic energy from a fixed point S on the axis of the spectrometer at a specified colatitude angle, θ_1. The "ultimate β-ray traces" are those corresponding to the trajectories which as the magnetic field is increased are the last of the entire pencil to be able to surmount the inner edge of the annular resolving slit.

The idea of the conical source is that, for β-rays of fixed energy E, when the magnetic field is set at just the value which permits only the ultimate ray from S to surmount the inner jaw edge at P (all other rays for angles θ both greater or less than θ_1 having been extinguished because of their inability to surmount the edge P) ultimate rays emanating from other

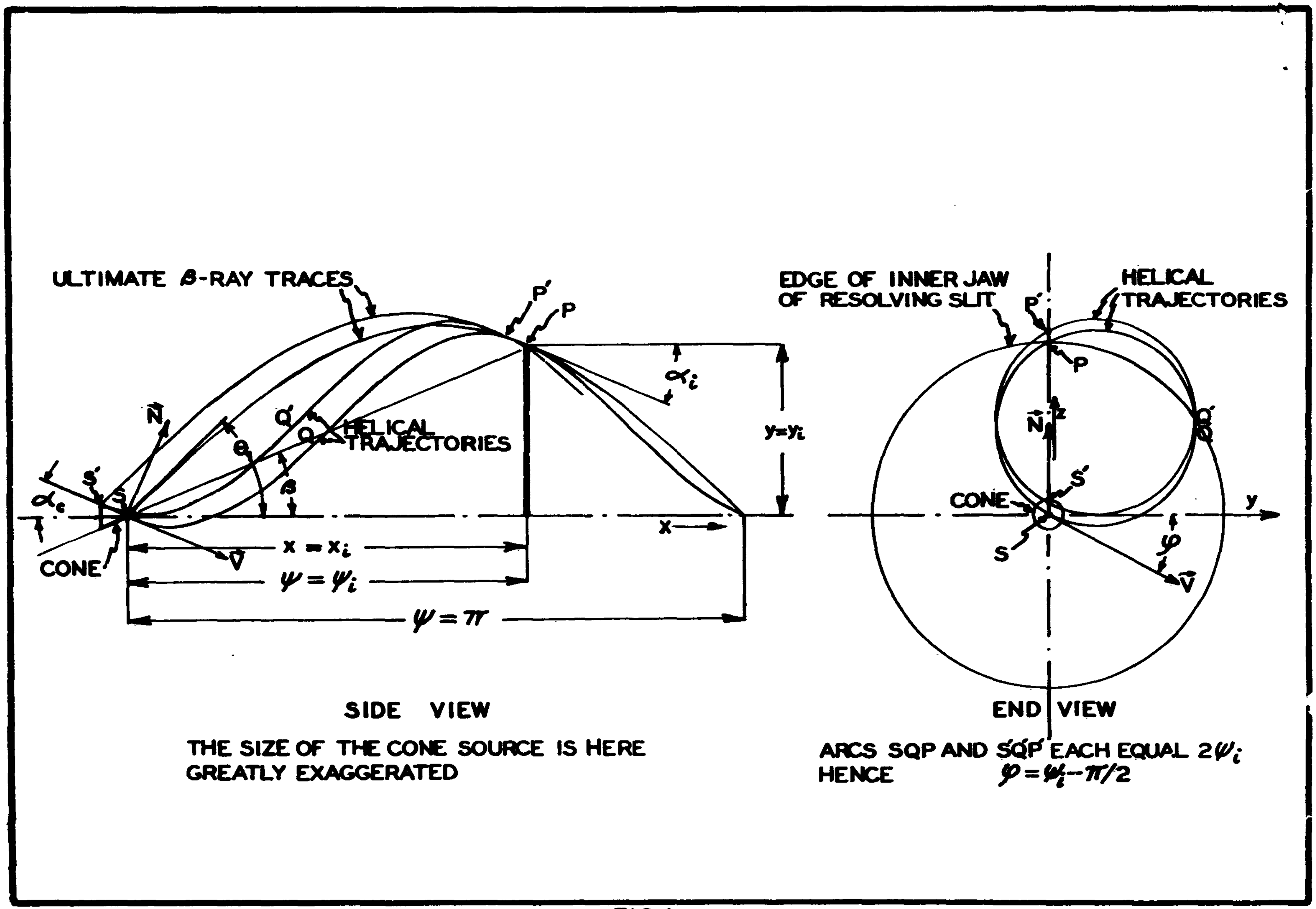
ULTIMATE β-RAY TRACES
P′
P
α_i
y=y_i
N
Q′
Q
HELICAL TRAJECTORIES
θ
β
S′
S
α_c
CONE
V
x
x = x_i
ψ = ψ_i
ψ = π
SIDE VIEW
THE SIZE OF THE CONE SOURCE IS HERE GREATLY EXAGGERATED
EDGE OF INNER JAW OF RESOLVING SLIT
HELICAL TRAJECTORIES
z
y
φ
END VIEW
ARCS SQP AND S′Q′P′ EACH EQUAL 2ψ_i
HENCE φ = ψ_i − π/2

FIG 1

points on the conical source will also still just be able to surmount the edge P provided the semi-apex angle of the cone α_o is made equal to α_1. For a point at a given azimuth of the inner jaw of the resolving slit (say the point P vertically above the axis as shown in the end view of Fig. 1) a whole element of the conical surface can supply ultimate β-rays through P or a point closely adjacent thereto such as P'. After an infinitesmally small further increase of the magnetic field however this entire element of the conical surface will be abruptly cut off. It was therefore predicted[1,2] that a quite sharp intercept of the leading edge of a β-ray conversion line with the horizontal background should be expected. This "Q-point" as it was called should, it was hoped, furnish an excellent fiducial point for determining the line energy.

The observed line profiles first obtained showed a leading edge which had quite a long straight segment with a fillet masking the expected "Q-point". It was at first believed that this fillet could be attributed to residual geometrical and magnetic imperfections in the instrument, and to natural breadths of the x-ray levels from which the β-rays came but we now have what appears to be fairly clear evidence that retardation of the ultimate β-rays in the source material also plays a part in creating this fillet so that the straight segment of the leading edge extrapolated to background does not faithfully give the "Q-point" for the energy of the line. We believe this can be attributed to the fact, which we shall proceed to prove, that for the ideal conical source geometry, the angle of emergence of the ultimate β-rays is always tangential or grazing with the surface element of the cone.

The angle β or the ratio of the coordinates x_1 and y_1 of the inner edge of the resolving slit relative to S determine the colatitude angle θ_1 for the ultimate ray and also the angle of inclination, α_1. Adopting the same symbols as those used in the authors first paper[3] on the theory of the optimum design of such a spectrometer it is easy to show from the equations (4) and (5) of that paper for x_1 and y_1

$$x_1 = (Q_1 \cos \theta_1)\psi_1$$

$$y_1 = (Q_1 \sin \theta_1) \sin \psi_1$$

that ψ_1 is given by the trancendental equation (7) of that paper

$$\tan^2 \theta_1 = - (\tan \psi_1 / \psi_1)$$

and that

$$\tan \beta = - \tan^2 \theta_1 \cos \psi_1 \qquad (1)$$

On the other hand Eq. (3) of the first paper, the equation of the ultimate trace itself is

$$y = Q \sin \theta_1 \sin \left[x/(Q \cos \theta_1) \right]$$

so that

$$-(\partial y / \partial x)_{x_1} = \tan \alpha_1 = - \tan \theta_1 \cos \psi_1 \qquad (2)$$

The end view in Fig. 1 shows the (circular) projection of an ultimate (helical) ray which passes from the point of the cone S through the point P vertically above the axis of the instrument on the inner jaw of the resolving slit. The same reasoning will apply to <u>any</u> point above S on an element of the cone lying in a vertical axial plane. We wish to find the cosine of the angle between a unit vector $\vec{V}$ tangent to the trajectory where it leaves the conical surface and a unit vector $\vec{N}$ normal to the element of the conical surface. The projection of the vector $\vec{V}$ shown in the end view Fig. 1 makes an azimuth angle $\varphi = \psi_1 - \frac{\pi}{2}$ with the horizontal y-coordinate and the vector $\vec{V}$ itself makes a latitude angle, $\frac{\pi}{2} - \theta_1$, with the (z,y)-plane. On a unit sphere of center S these two last mentioned angles are the legs of a right spherical triangle whose hypothenuse angle β_1 gives the angle between $\vec{V}$ and the y-axis. From this right triangle we have by the law of cosines

$$\cos \beta_1 = \cos\left(\psi_1 - \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - \theta_1\right) = \sin \psi_1 \sin \theta_1 \qquad (3)$$

We are now in a position to write the direction cosines of $\vec{V}$ and of $\vec{N}$ with respect to the x, y and z axes. These we tabulate below.

	x	y	z
Direction cosines of $\vec{N}$	$\sin \alpha_c$	0	$\cos \alpha_c$
Direction cosines of $\vec{V}$	$\cos \theta_1$	$\sin \theta_1 \sin \psi_1$	$\sin \theta_1 \cos \psi_1$

It follows therefore that the cosine of the angle between $\vec{N}$ and $\vec{V}$ is

$$\cos(\vec{N}\vec{V}) = \sin \alpha_c \cos \theta_1 + \cos \alpha_c \sin \theta_1 \cos \psi_1 \qquad (4)$$

Now if we make $\alpha_c = \alpha_1$ to take best advantage of Muller's idea we must have from (2)

$$\tan \alpha_c = - \tan \theta_1 \cos \psi_1 \qquad (5)$$

We are at liberty to abandon strict adherence to the rule of choosing θ_1 close to 45° (where the optimum condition for highest resolving power for a given utilized solid angle in the pencil of β-rays obtains). The very flat minimum in the curve of Fig. 6 in reference (3) means that no great sacrifice is made anywhere in the range $30^\circ < \theta_1 < 60^\circ$. We shall therefore regard θ_1 as a variable. Form (3) we see that the factor $\cos \alpha_c \cos \psi_1$ of the second term in (4) can be written

$$\cos \alpha_c \cos \psi_1 = - \sin \alpha_c / \tan \theta_1 \qquad (6)$$

and substituting this into (4) we see that $\cos \vec{N}\vec{V}$ <u>vanishes identically</u> for all values of θ_1 and corresponding α_c, which means that so long as we adhere to the rule $\alpha_c = \alpha_1$ the ultimate β-rays must leave the surface of the cone at grazing emergence independent of what choice is made for θ_1.

We are thus in the dilemma that if we make $\alpha_c = \alpha_i$ we shall have large retardation even close to the foot of the leading edge of the line because of grazing emergence and if we make $\alpha_c > \alpha_i$ to avoid grazing emergence we shall have a less abrupt termination at the foot of the leading edge of the line because different portions of the cone further and further from the apex will be cut off progressively as the magnetic field is increased.

In a preliminary experimental study conducted by Dr. Pierre Marmier and Dr. James Wilts the line structure obtained with exactly the same line (Th "I"-line, 238 kev level in Bi^{212} Th C) for three different source geometries was compared and the results are shown in Fig. 2. Curve (1) is made with a conical source, the thorium being deposited on a small aluminum cone of semi-apex angle 23° 48' and of base diameter 1.3 mm from the vapor state by the well known method using an electric field. This method tends to concentrate the deposit close to the point by reason of the concentration of the field and is therefore especially favorable to the conical source as compared to the two other sources. Curve (2) is made with a source deposited in the shape of a disk with approximately uniform surface density on an aluminum support of substantially infinite thickness (for the purposes of back scattering). The source disk was centered on and normal to the axis of the spectrometer and was of the same diameter (1.3 mm) as the base of the cone source. Its axial position was adjusted with great care to be exactly the same relative to the resolving slit as that previously occupied by the point of the cone source. Curve (3) was made with a disk source of the same diameter as that of Curve (2) but deposited on an extremely thin Zapon backing. In this case also the axial position of the center of the source was carefully adjusted to coincide with that previously occupied by

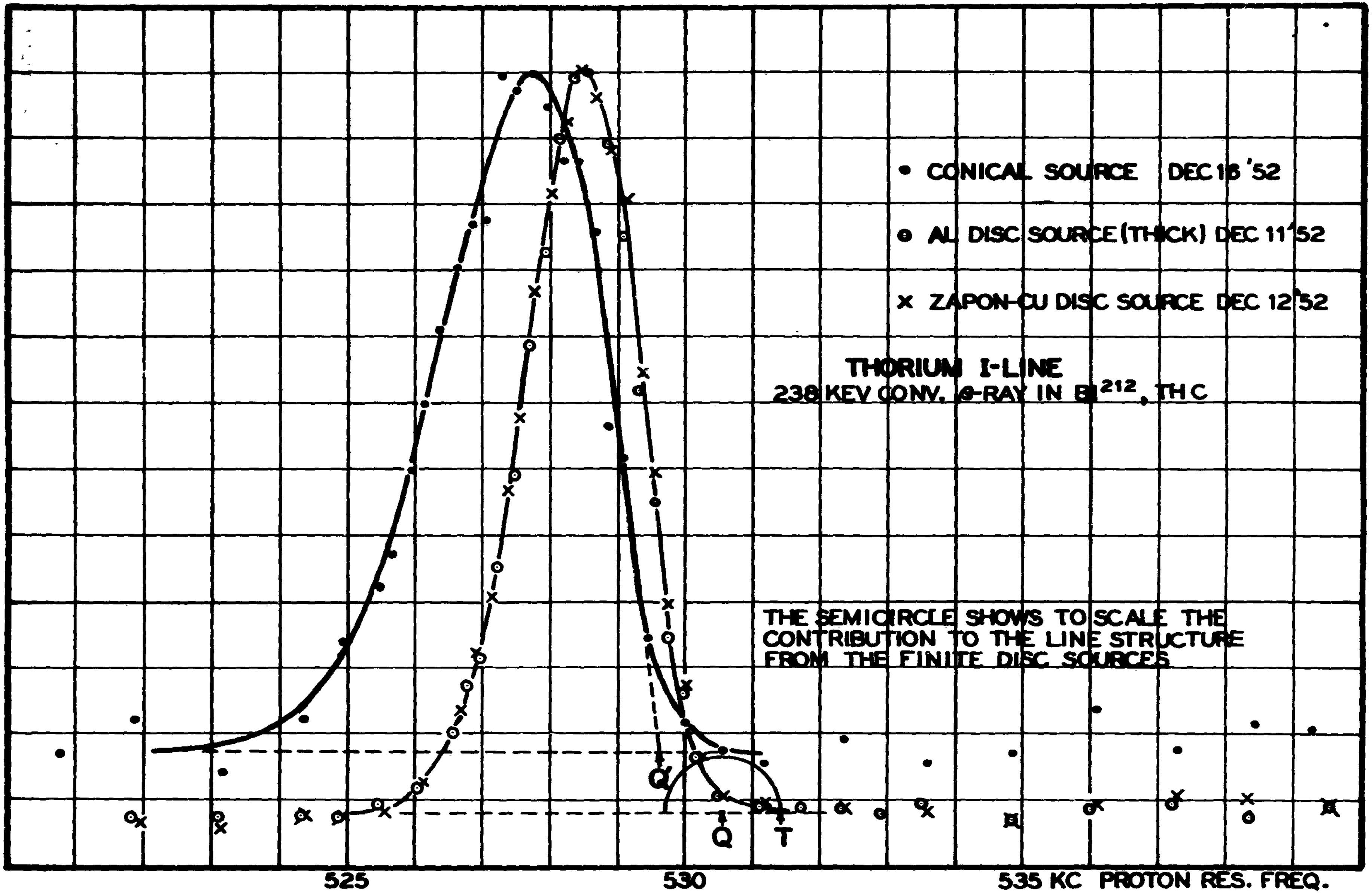
• CONICAL SOURCE DEC 18 '52
⊙ AL DISC SOURCE (THICK) DEC 11 '52
× ZAPON-CU DISC SOURCE DEC 12 '52
THORIUM I-LINE
238 KEV CONV. γ-RAY IN Bi212, TH C
THE SEMICIRCLE SHOWS TO SCALE THE CONTRIBUTION TO THE LINE STRUCTURE FROM THE FINITE DISC SOURCES
Q
Q
T
525
530
535 KC PROTON RES. FREQ.

FIG 2

the point of the cone.

The straight segment of the leading edge of the profile made with the cone source when extrapolated to the background level yields the point marked Q' as shown. It would be incorrect to extrapolate the straight segment of the leading edge of the profile made with either disc source to background for comparison with Q'. Such a procedure would only be admissable if the profiles made with the disc sources were first corrected to what they would have been if made with an infinitesmally small disc. This would require a rather elaborate solution of a complicated integral equation which the roughness of the present data would hardly justify. From the known radius, p = 0.65 mm, of the disc, however, it is easy to calculate from formulae (30) and (31) of reference 3 that the half-width A_3 of the elliptical contribution to the line profile (Fig. 7 III, p. 166 of Ref. 3) corresponds to a fractional change in electron momentum $\Delta p/p$ (or in magnetic field intensity $\Delta B/B$) equal to $\rho/(2R)$ where R is the maximum radial departure of the ultimate β-ray trajectories from the axis of the instrument. In the present instrument R = 200 mm and it is easy therefore to calculate that at a proton resonance frequency of 530 kc the half width A_3 of the elliptical profile resulting from the finite disc source should be 0.86 kc. The true "Q-point" should therefore be about 0.86 kc to the left of the point T where the fillet of the disc source profiles meets (becomes tangent to) the background for that profile. This seems to call for a point Q at least 0.5 kc to the right of Q'. This argument is admittedly rough since it is based on the assumption of unfform source distribution over the disc sources. The location of the tangent point T is also uncertain. Under no circumstance, however, does it seem plausible to us in the light of these data to identify Q' as the true "Q-point" in the sense first anticipated for the theory of the conical sources.

A more important consideration still is the following. The three profiles in Fig. 2 have been normalized to about the same peak height and it will be noted that when this has been done both flat sources yield a steeper leading edge in the straight segment of the latter than the cone source with a much more restricted fillet where the leading edge meets the background. Very striking also, and we believe of great utility, is the fact that for this internal conversion line the profiles from both flat sources whether with thick or thin backing are, as nearly as can be judged, identical in shape though they differ markedly from the profile given by the cone source. They are also considerably narrower than the latter.

The conclusion seems to be that for β-rays in this range of energies the advantages which we at first attributed to the conical sources do not exist and that this is in all probability due to the tangential or grazing emergence of the β-rays from the surface elements of such sources. It seems almost certain that even in the absence of all imperfections in instrument geometry or homogeneity of field and even with perfectly sharp x-ray energy levels from which the internally converted β-rays originate, the characteristic leading edge profile for the cone source must meet the background not with a sharply defined intersection but with a fillet making rather high order contact with the background because of the retardation of the β-rays coming from the surface of the cone at grazing emergence.

A more promising idea which is under investigation for the determination of conversion line energies to high precision is to abandon any effort to identify some particular "Q-point" on the line profile and to obtain the momentum (or energy) difference between two lines of different energy, one of which is a calibration line and the other and unknown line, by a method of superposing the profiles after appropriate coordinate transformations which cause the two profiles to match closely in shape. For this purpose it appears at present very promising merely to plot all lines normalized to the same peak height against the logarithm of the field strength (or the logarithm of the proton resonance frequency). (It goes without saying that the background is first to be subtracted off). This device is under careful investigation at present, using small disc sources.

The fact that all single line profiles, so plotted, over a considerable range of energies exhibit the same profile also offers the attractive possibility of detecting the presence of close unresolved multiplets which would be revealed by distortion of the standard profile.

LIST OF REFERENCES

1) An Axial Focusing Magnetic β-Ray Spectrometer of High Luminosity, Resolving Power and Precision With Proton-Resonance-Stabilized Homogeneous Field, A Report to the Research Corporation, also Special Technical Report No. 16, Contract N6onr-244, T.O. IV, J.W.M. DuMond, L. Bogart, J.L. Kohl, D.E. Muller and J.R. Wilts. Contains an exposition of the conical source idea.

2) Improvements in the Precision of Beta-Ray Spectroscopy, J.W.M. DuMond, "Physics Today" 5, No. 12, 10 (Dec. 1952). Also contains brief exposition of conical source idea.

3) Jesse W.M. DuMond, Rev. Sci. Inst. 20, 160 (1949)

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To Recipients of Our Earlier Reports on the
Homogeneous Field Precision β-Ray Spectrometer.

A report dated March 1952, and entitled "An Axial Focusing Magnetic Beta-Ray Spectrometer of High Luminosity, Resolving Power and Precision with Proton-Resonance-Stabilized Homogeneous Field - Without Iron" (prepared both as a final report to the Research Corporation, donors of the funds for construction of the instrument and also as Special Technical Report No. 16 to the Office of Naval Research) described the new instrument in considerable detail. At the time that report was prepared D. E. Muller's idea of using conical shaped β-ray sources had just been tried for the first time and from our preliminary measurements made with rather crude instrumentation, both for β-ray counting and for measuring the proton resonance frequencies, we became very enthusiastic about this method because of the apparent possibility it seemed to offer of giving a reliable fiducial point ("Q-point") on the line profile to which a definite absolute meaning related to the β-ray momenta could be attached.

In an article in the Dec. 1952 issue of "Physics Today" the idea of Muller's conical sources was again described and the same favorable preliminary results as those obtained earlier in the year were presented. Shortly after the appearance of this article however thanks to several improvements in sensitivity and reduction of background more accurate evidence was obtained by P. Marmier and J. Wilts working with this instrument which has cast grave doubt on the absolute reliability of the fiducial "Q-point" from the conical sources.

These more careful investigations have now shaken our earlier confidence in this method as regards its reliability for really high absolute precision for reasons set forth in the accompanying mimeographed preprint which we hasten to distribute to prevent others from being misled into possibly expensive and time-consuming experiments. The difficulty seems to stem from the fact, proven in the accompanying note, that the β-rays for the case of the conical source, emerge from the surface elements of the cone <u>at a grazing angle in all cases</u>.

An alternative method which does not call for the selection of any one particular fiducial point on the β-ray line structure is now under careful study using simple disc shaped sources and this promises to have several additional advantages over the conical-source-and-"Q-point" method. We contemplate publishing all this as a note or article in the physical journals as soon as we have sufficient constructive material to present. The purpose of the present <u>preprint</u> is merely therefore to correct promptly any misimpression that our <u>earlier</u> favorable reports about the conical sources may have created in the minds of those who might otherwise be misled by them.

Sincerely yours,

Jesse W.M. DuMond

Jesse W.M. DuMond